JEE (Main)-2025 (Online) Session-2

4 April 2025 Shift – 1

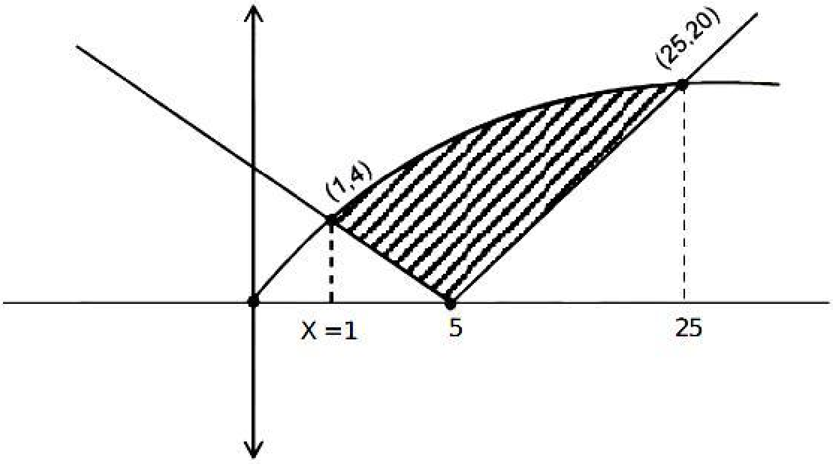
PART : MATHEMATICS

1. Area bounded by curves $|x-5| < y \le 4\sqrt{x}$ is A, then the value of 3A is equal to

(1) 368 (2) 81 (3) 225 (4) 96

Ans. **(1)**

Sol.



$y = x - 5$ & $y = 4\sqrt{x}$

$4\sqrt{x} = x - 5$

$x - 4\sqrt{x} - 5 = 0$

$\sqrt{x} = 5,\ \sqrt{x} = -1$

$x = 25$

Solving $y = 5 - x$ & $y = 4\sqrt{x}$

$x + 4\sqrt{x} - 5 = 0$

$\sqrt{x} = 1 \quad \Rightarrow x = 1$

Required area

$= \int_1^{25} 4\sqrt{x}\,dx - \frac{1}{2} \times 4 \times 4 - \frac{1}{2} \times 20 \times 20$

$= \left(4.x^{3/2}.\frac{2}{3}\right)_1^{25} - 8 - 200$

$= \frac{8}{3}(25 \times 5 - 1] - 208$

$= \frac{8}{3} \times 124 - 208$

$A = \frac{992 - 624}{3}$

Now $3A = 368$

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2. In the expansion of $\left(2^{\frac{1}{3}}+3^{-\frac{1}{3}}\right)^n$, if $\frac{T_{15} \text{ from begining}}{T_{15} \text{ from end}}=\frac{1}{6}$, then the value of ${}^nC_3$ is equal to :

(1) 2200 (2) 2300 (3) 2400 (4) none of these

Ans. **(2)**

Sol. $\therefore$ T_{15} from beginning = ${}^nC_{14}\left(2^{\frac{1}{3}}\right)^{n-14}\left(3^{-\frac{1}{3}}\right)^{14}$

T_{15} from end = ${}^nC_{14}\left(3^{-\frac{1}{3}}\right)^{n-14}\left(2^{\frac{1}{3}}\right)^{14}$

$$\frac{{}^nC_{14}2^{\frac{n-14}{3}}.3^{-\frac{14}{3}}}{{}^nC_{14}2^{\frac{14}{3}}.3^{-\frac{(n-14)}{3}}}=\frac{1}{6}$$

$$2^{\frac{n-28}{3}}.3^{\frac{n-28}{3}}=\frac{1}{6}=6^{-1}$$

$$\frac{n-28}{3}=-1 \qquad n=25$$

$\therefore$ ${}^nC_3={}^{25}C_3=\frac{25\times24\times23}{3\times2\times1}=2300$

3. The sum of 40 terms of $1^2+3+5^2+7+9^2+ \ldots\ldots$ is equal to

(1) 39600 (2) 41880 (3) 43500 (4) 38500

Ans. **(2)**

Sol. $\sum_{r=1}^{20}(4r-3)^2+\sum_{r=1}^{20}(4r-1)$

$=\sum(16r^2-24r+9+4r-1)$

$=\sum_{r=1}^{20}(16r^2-20r+8)$

$=\frac{16\times20\times21\times41}{6}-\frac{20\times20\times21}{2}+160$

$=16\times70\times41-10\times420+160$

$=41880$

4. If roots of $x^2-4x-n=0$ are integral, $n\in N$ and $n\in[20, 100]$ then number of values of 'n'

Ans. **(6)**

Sol. D = Perfect square

$16+4n=\ell^2$

$n=\frac{\ell^2-16}{4}$

ℓ	n
6	5
8	12
10	21

Values of n are 21, 32, 45, 60, 77, 86

So total values of n = 6

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5. If the arithmetic mean of binomial coefficient in the expansion of $(x+y)^{2n-3}$ is equal to 16, then the length of perpendicular drawn from the point $(2n-1, n^2-4n)$ to the line $x+y=8$, is equal to

(1) $3\sqrt{2}$ (2) $4\sqrt{2}$ (3) $5\sqrt{2}$ (4) $6\sqrt{2}$

Ans. **(1)**

Sol. $\therefore$ $\dfrac{^{2n-3}C_0 + {}^{2n-3}C_1 + {}^{2n-3}C_2 + ... + {}^{2n-3}C_{2n-3}}{2n-2} = 16$.

$2^{2n-3} = 16(2n-2) = 2^5(n-1)$

$\Rightarrow$ $2^{2n-8} = (n-1)$

$\Rightarrow$ $n = 5$

$\therefore$ $(2n-1, n^2-4n) \equiv (9, 5)$

$\therefore$ Required length $= \left|\dfrac{9+5-8}{\sqrt{1+1}}\right| = 3\sqrt{2}$

6. Let $10\sin^4\theta + 15\cos^4\theta = 6$

Then value of

$\dfrac{27\operatorname{cosec}^6\theta + 8\sec^6\theta}{16\sec^8\theta}$ is equal to

(1) $\frac{3}{5}$ (2) $\frac{2}{5}$ (3) $\frac{35}{16}$ (4) $\frac{19}{16}$

Ans. **(2)**

Sol. Let $\sin^2\theta = t$

$10t^2 + 15(1-t)^2 = 6$

$25t^2 - 30t + 9 = 0$

$t = \frac{3}{5}$

$\sin^2\theta = \frac{3}{5}$

$\cos^2\theta = \frac{2}{5}$

Now

$\dfrac{27\operatorname{cosec}^6\theta + 8\sec^6\theta}{16\sec^8\theta}$

$= \dfrac{27\left(\frac{125}{27}\right) + 8\left(\frac{125}{8}\right)}{16 \times \frac{625}{16}}$

$= \dfrac{125+125}{625} = \dfrac{10}{25} \Rightarrow \dfrac{2}{5}$

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7. The value of $\int_{-1}^{1}\left(\frac{\left(1+\sqrt{|x|-x}\right)e^{x}+\left(\sqrt{|x|-x}\right)e^{-x}}{e^{x}+e^{-x}}\right)dx$ is equal to

(1) $2-\frac{\sqrt{2}}{3}$ (2) $2+\frac{2\sqrt{2}}{3}$ (3) $1+\frac{2\sqrt{2}}{3}$ (4) $1+\frac{\sqrt{2}}{3}$

Ans. **(3)**

Sol. Using $\int_{-a}^{a}f(x)dx=\int_{0}^{a}f(x)+f(-x)dx$, we obtain

Given integral

$$=\int_{0}^{1}\frac{\left(1+\sqrt{|x|-x}\right)e^{x}+\left(\sqrt{|x|-x}\right)e^{-x}+\left(1+\sqrt{|x|+x}\right)e^{-x}+\left(\sqrt{|x|+x}\right)e^{x}}{e^{x}+e^{-x}}dx$$

$$=\int_{0}^{1}\frac{e^{x}+e^{-x}+\left(\sqrt{|x|-x}\right)\left(e^{x}+e^{-x}\right)+\left(\sqrt{|x|+x}\right)\left(e^{-x}+e^{x}\right)}{e^{x}+e^{-x}}dx$$

$$=\int_{0}^{1}\left(1+\sqrt{|x|-x}+\sqrt{|x|+x}\right)dx=\int_{0}^{1}(1+\sqrt{2x})dx$$

$$=1+\sqrt{2}.\left[\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right]_{0}^{1}=1+\frac{2\sqrt{2}}{3}$$

8. If two foci of an ellipse are (2, 5) and (2, –3) and its e = 4/5, then the length of latus rectum of this ellipse is equal to

(1) $\frac{18}{5}$ (2) $\frac{9}{5}$ (3) $\frac{16}{5}$ (4) $\frac{12}{5}$

Ans. **(1)**

Sol. $\therefore\quad 2ae=\sqrt{64}=8$

$\Rightarrow\quad ae=4 \qquad \Rightarrow a=5 \quad$ (as e = 4/5)

$\because\quad ae=4$

$a^2-b^2=16 \qquad \Rightarrow b^2=9$

$\therefore\quad \ell\,(L.R)=\frac{2b^2}{a}=\frac{18}{5}$

9. Let $A=\{1, 6, 11, 16, \ldots\ \text{upto } 2025 \text{ terms}\}$ & $B=\{9, 16, 23, \ldots\ 2025 \text{ terms}\}$ then $n(A\cup B)$ is ____.

(1) 3761 (2) 3650 (3) 3810 (4) 3619

Ans. **(1)**

Sol. Common first term = 16

and common difference of common term be 35

Last term of set A is $=1+2024\times 5$

$=10121$

Last term of set B $=9+2024\times 9$

$=14168+9$

$=14177$

Now Let number of common term be n.

$$16+(n-1)\times 35\le 10121$$
$$35n-19\le 10121$$
$$35n\le 10140$$
$$n\le 289.7$$

So 289 terms are common

Now $n(A\cup B)=2025+2025-289=3761$

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10. There are 10 pens in which 3 are defective. 2 samples of pen are selected randomly. Let x be number of defective pen, then variance of probability distribution of number of defective pen is equal to

(1) $\frac{28}{75}$ (2) $\frac{37}{75}$ (3) $\frac{26}{75}$ (4) $\frac{23}{75}$

Ans. **(1)**

Sol.

x	0	1	2
P(x)	$\frac{{}^{7}C_2}{{}^{10}C_2}$	$\frac{{}^{7}C_1 \times {}^{3}C_1}{{}^{10}C_2}$	$\frac{{}^{3}C_2}{{}^{10}C_2}$

$$\text{Variance} = \sum P_i(x_i)^2 - \left(\sum P_i x_i\right)^2$$

$$= \left(0 + \frac{21}{45} \times 1^2 + \frac{3}{45} \times 2^2\right) - \left(0 + \frac{21}{45} \times 1 + \frac{3}{45} \times 2\right)^2$$

$$= \frac{33}{45} - \left(\frac{27}{45}\right)^2$$

$$= \frac{11}{15} - \left(\frac{3}{5}\right)^2$$

$$= \frac{55 - 27}{75}$$

$$= \frac{28}{75}$$

11. A committee of 12 members is formed randomly out of 4 Engineers, 2 Doctors and 10 Professors . Find the probability that the committee has atleast 3 Engineers and atleast 1 Doctor.

(1) $\frac{103}{182}$ (2) $\frac{129}{182}$ (3) $\frac{107}{182}$ (4) $\frac{109}{182}$

Ans. **(2)**

Sol. Engineers = 4, Doctors = 2, Professors = 10

$n(S) = {}^{16}C_{12} = {}^{16}C_4 = 14 \times 13 \times 10$

$\therefore$ n(A) =

Eng.	Dr.	Pr.	
3	1	8	$= {}^{4}C_3 \times {}^{2}C_1 \times {}^{10}C_8 = 4 \times 2 \times 45 = 360$
4	1	7	$= {}^{4}C_4 \times {}^{2}C_1 \times {}^{10}C_7 = 1 \times 2 \times 120 = 240$
3	2	7	$= {}^{4}C_3 \times {}^{2}C_2 \times {}^{10}C_7 = 4 \times 1 \times 120 = 480$
4	2	6	$= {}^{4}C_4 \times {}^{2}C_2 \times {}^{10}C_6 = 1 \times 1 \times 210 = 210$

$\therefore$ $n(A) = 1290$

$\therefore$ $P(\text{Reqd.}) = \frac{n(A)}{n(S)} = \frac{1290}{14 \times 13 \times 10} = \frac{129}{14 \times 13} = \frac{129}{182}$

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12. If $\lim_{x\to 1^+}\frac{(1-x)(1+\lambda\cos(x-1))+\mu\sin(1-x)}{(1-x)^3}=-1$, then $\lambda+\mu$ is equal to

(1) –1 (2) 1 (3) 2 (4) 3

Ans. **(1)**

Sol. $\lim_{h\to 0}\frac{(-h)[1+\lambda\cosh]+\mu(\sin(-h))}{-h^3}$

$$=\lim_{h\to 0}\frac{-h\left[1+\lambda\left(1-\frac{h^2}{2!}+\ldots\right)\right]+\mu\left(-h+\frac{h^3}{3!}\right)}{-h^3}$$

Now $\frac{-h(1+\lambda+\mu)+h^3\left(\frac{\lambda}{2}+\frac{\mu}{6}\right)}{-h^3}=-1$

Now $\lambda+\mu=-1$ and $-\frac{\lambda}{2}-\frac{\mu}{6}=-1$

Now $\lambda+\mu=-1$

13. If $A=\begin{bmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{bmatrix}$, such that $A^2=A^T$, then find trace of $((A+I)^3+(A-I)^3-6A)$

Ans. **(6)**

Sol. $A^2=A^T\Rightarrow$ $\begin{bmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{bmatrix}=\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}$

$\Rightarrow$ $\begin{bmatrix}\cos2\theta & 0 & -\sin2\theta\\ 0 & 1 & 0\\ \sin2\theta & 0 & \cos2\theta\end{bmatrix}=\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}$

$\Rightarrow$ $\cos2\theta=\cos\theta$, $\sin2\theta=-\sin\theta\Rightarrow 2\cos^2\theta-\cos\theta-1=0$, $\sin\theta(2\cos\theta+1)=0$

$\Rightarrow$ $(\cos\theta-1)(2\cos\theta+1)=0$, $\sin\theta(2\cos\theta+1)=0$

$\Rightarrow$ $\cos\theta=1$, $\sin\theta=0$ or $\cos\theta=\frac{-1}{2}$

$\Rightarrow$ $\theta=2n\pi$ or $\theta=2m\pi\pm\frac{2\pi}{3}$

$A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ or $A=\begin{bmatrix}\frac{-1}{2} & 0 & \frac{-\sqrt{3}}{2}\\ 0 & 1 & 0\\ \frac{\sqrt{3}}{2} & 0 & \frac{-1}{2}\end{bmatrix}$ or $A=\begin{bmatrix}\frac{-1}{2} & 0 & \frac{\sqrt{3}}{2}\\ 0 & 1 & 0\\ \frac{\sqrt{3}}{2} & 0 & \frac{-1}{2}\end{bmatrix}$

$\Rightarrow$ $AA^T=I\Rightarrow A(A^2)=AA^T\Rightarrow A^3=I$

$(A+I)^3=A^3+3A^2+3A+I$

$(A-I)^3=A^3-3A^2+3A-I$

$((A+I)^3+(A-I)^3-6A)=2A^3=2I\Rightarrow$ Trace $=6$

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